Production analytics based on dynamical symmetry

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Outline

1. Introduction
2. The Physical Setting
3. Relations
4. Example: Tight Gas Well
5. Concluding Remarks

What To Do With Production Analytics ?

- Given a collection of production analytics tools and some data
- We could start processing the data with the tools, do computations
- But whatever the computational results might be, what do they mean, what is their **interpretation**
- $\Longrightarrow$ Along this route, defining the problem by the production analytics tools, it is unclear what to do with them
- $\Longrightarrow$ We try the reverse route: defining the production analytics tools by the problem

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A Production System

- Given a *producing* production system $\mathscr{S}$ - like a well, or a group of wells -
- Given a set of *measurements* , like pressure, cumulative production, production rate, temperature, and all that, as follows: $\{X^j_{t_i} \in \mathbb{R} | t_i \in \mathbb{R}, i \in \{1, \ldots, n\}; j \in \{1, \ldots, m\}\}$ ordered in time $(t_1 < \cdots < t_n)$ and possibly recorded at different locations in $\mathscr{S}$ - so $X^{j_1}_{t_1}$ and $X^{j_2}_{t_1}$ may both be pressures, recorded at time t_1, and the first one is a down-hole pressure, and the second one a surface tubing pressure.

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Measurement '=' Assigning A Number To An Object

Definition

The system $\mathscr{S}$ may be thought of to be described by a collection of *theoretical* - in the sense that we have no direct 'access' to them - physical quantities $\{Y^j | \, j \in \{1, \ldots, k\}, k > m\}$, where, for the sake of definiteness, it is assumed that $Y^j \in \mathbf{C}^\infty([t_1, t_n])$, and denoted by $Y^j(t)$. The measurement $X_{t_i}^{j_k}$ is the value - *approximately* - assigned to $Y^{j_k}(t)$ when evaluated for $t = t_i$; it describes in this way some quality or aspect of the abstract physical quantity Y^{j_k}, and hence of the system $\mathscr{S}$.

- Because of *noise*, $X_{t_i}^{j_k}$ may only be approximately equal to $Y^{j_k}(t)_{|_{t=t_i}}$; they have the same physical meaning

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$\mathscr{S}$ is a succession of *causes* and *effects*

- There are relations among all physical quantities of $\mathscr{S}$. Let $0_{\mathbf{C}([t_1,t_n])}$ be the zero of $\mathbf{C}([t_1,t_n])$, then, with $f:\mathbf{C}([t_1,t_n]) \to \mathbf{C}([t_1,t_n])$, here is an example of such a relation:
  $f(Y^{j_1}(t),\ldots,Y^{j_l}(t)) = 0_{\mathbf{C}([t_1,t_n])} \quad (\{j_1,\ldots,j_l\} \subseteq \{1,\ldots,k\})$
- The above relation may very well be a differential equation (think of $Y^{j_2}(t) = dY^{j_1}(t)/dt$, or more specifically Y^{j_1} represents the cumulative production of, say, a tight gas well, and Y^{j_2} the production rate of that well)
- These relations must obey the governing physical principles

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Contact With Reality

- The relations among all physical quantities induce relations among the **measured** physical quantities through a process called **natural elimination**
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- The above relation is called an **In-Situ Physical Law**

Pointwise version:

$$g(Y^{i_1}(t_l),\ldots,Y^{i_p}(t_l)) = 0 \quad (\{i_1,\ldots,i_p\} \subseteq \{1,\ldots,m\}\ ;\ l \in \{1,\ldots,n\})$$

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Confrontation with Reality

Rather than pondering further about what this all might mean, and what we could do with it, we must first of all face a very urgent matter:

How To Obtain, in any case To Approximate the In-Situ Physical Laws

- they may be non-linear differential - or algebraic equations
- our only entrance is the measured data
- the connection *physical quantities* $\longleftrightarrow$ *measured data* does not work

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Approximating The In-Situ Physical Laws

- the desired connection *Measured Data* $\longrightarrow$ *In-Situ Physical Laws* does not exist
- $\Rightarrow$ we are looking for objects that can be reached from the data, and can be connected to, in any case be in the 'domain' of the In-Situ Laws

Employing A Mathematical Freedom

Consider the measurements as evaluations of an abstract object having the desired properties

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The Abstract Associate Of A Measurement

- Consider the differential polynomial ring $\mathbf{P} = \mathbb{R}\{x^1, \ldots, x^m\}$ generated by the differential indeterminates $\{x^j | j = 1, \ldots, m\}$
- Consider $eval_{t_i} : \mathbf{P} \to \mathbb{R}$ defined by $x^j \mapsto X^j_{t_i}$
- the ring structure of $\mathbf{P}$ takes care of the following
  - non-linearities: we need **not** introduce non-linearities
  - $\mathbf{P}$ has a derivation, denoted by ∂_t
- the results from $\mathbf{P}$, notably relations in $\mathbf{P}$, can be translated to $\mathbf{C}^\infty([t_1, t_n])$ (viewed as a ring): $\mathbf{P} \xrightarrow{\Phi} \mathbf{C}^\infty([t_1, t_n])$; in particular $\partial_t \to d/dt$

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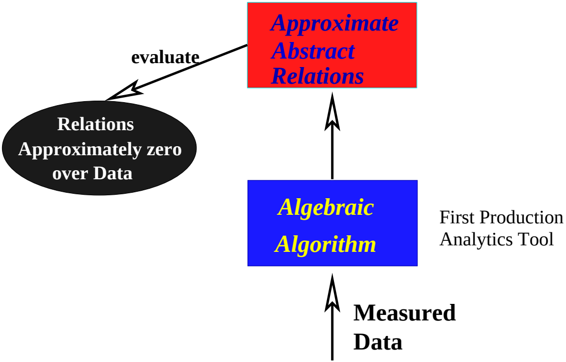
From Data To Abstract



- let $p(x^{i_1}, \ldots, x^{i_q}) \approx 0_{\mathbf{P}}$, i.e. $p(X_{t_j}^{i_1}, \ldots, X_{t_j}^{i_q}) \approx 0$ $(j = 1, \ldots, n)$, then p is an approximately vanishing differential polynomial, i.e. an approximate, abstract relation

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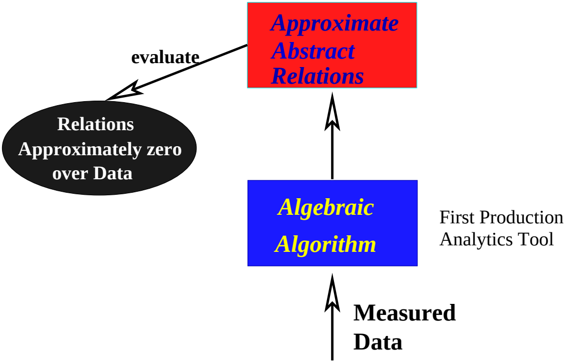

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- its pointwise version is: $p(\hat{Y}^{i_1}(t_i),\ldots,\hat{Y}^{i_q}(t_i)) \approx 0 \; (i=1,\ldots,n)$
- if the data $\{\hat{Y}^{i_1}(t_i),\ldots,\hat{Y}^{i_q}(t_i)\}_{i=1}^{n}$ satisfying this approximate relation can be validated against the measured data $\{X_{t_i}^{i_1},\ldots,X_{t_i}^{i_q}\}_{i=1}^{n}$, it is called an **Approximate In-Situ Physical Law**

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A Quick Aside

- Research is on-going to find the 'associated' collection of exactly vanishing polynomials from the collection of approximately vanishing polynomials along a **physical-algebraic** route
- This would recover the (exact) In-Situ Physical Laws
- This is very different from, or rather has nothing to do with traditional noise reduction/averaging techniques
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- although we made our first choice of a production analytics tool, it is still too early for sensible computations
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The Perspective - simple example

- the geometry in the given connection is determined by the choice of the considered quantities
- suppose we consider $\mathscr{S}$ in terms of the quantities $\{X, dX/dt, t\}$
  - this is looking at $\mathscr{S}$ in the *contact geometry* ; for a production problem this is 'looking through the wrong glasses', and so it will be hard to interpret the relations - but not for exploration problems: take X the amplitude at a certain location and we are in the realm of the Huygens principle

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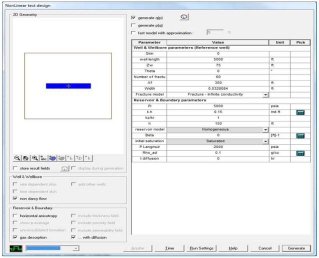

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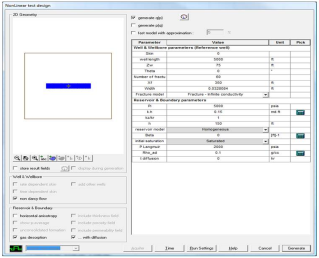

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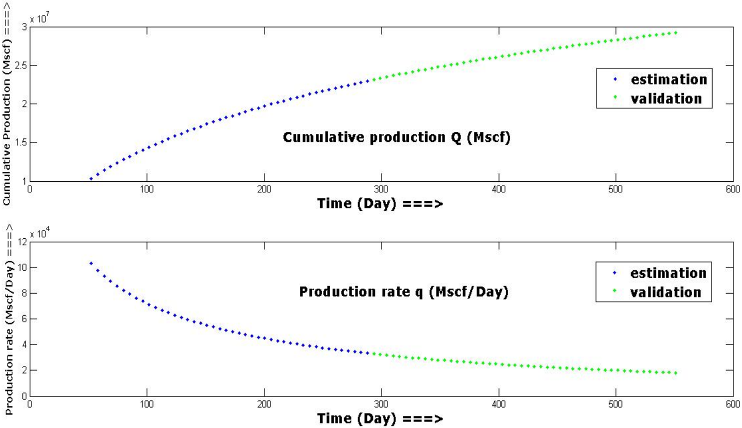
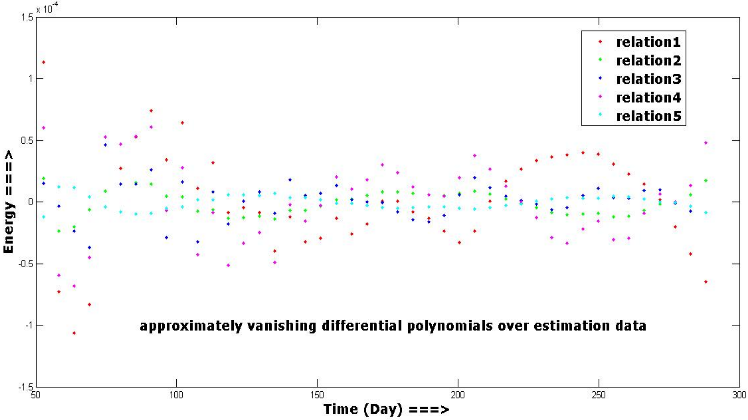


Approximately Vanishing Relations



The Converse of a famous Principle

- **D' Alembert's Principle**: (in reformulating Newton's law) there is a shift from '***dynamics to statics or equilibrium***'
- In the current situation the converse of this principle holds:
  - There is a shift from equilibrium - the vanishing relations - to dynamics
  - More specifically the vanishing relations can be decomposed into two non-vanishing relations
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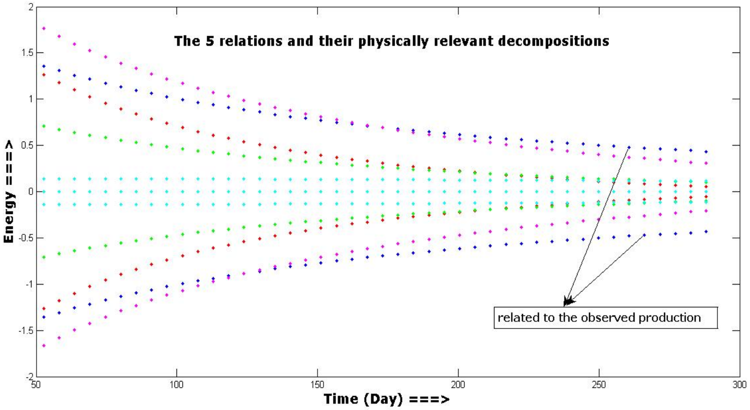
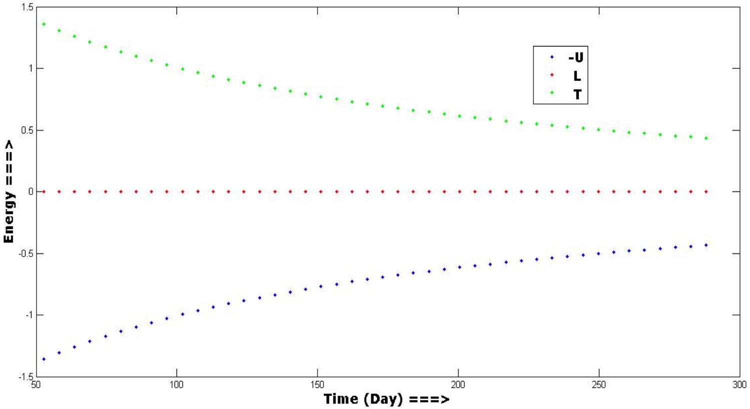
The Dynamical Symmetry Portrait



The Flow Relation



The Flow Relation

- the flow relation is chosen from considerations of the *Dynamical Symmetry Portrait*

- T is interpreted as the kinetic energy, the energy related to the flow of the gas through the reservoir
  - $T = c_1 *^3 + c_2 * q^2 + c_3 * Q^2 * q + c_4 * Q * q \quad (c_i \in \mathbb{R})$
  - the coefficients c_i depend on the physical parameters of $\mathscr{S}$
  - because the physical dimension of T is that of an energy, the physical dimension of the c_i is known
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- U is interpreted as the *push from the reservoir*
- there is in the tight gas problem of course also work done by a force which is not derived from a potential (and so $T + U$ is not constant)
- from this information an *equation of motion* can be derived (a second-order, non-linear differential equation in Q), describing the flow of the gas through the reservoir
- the equation of motion can be used for
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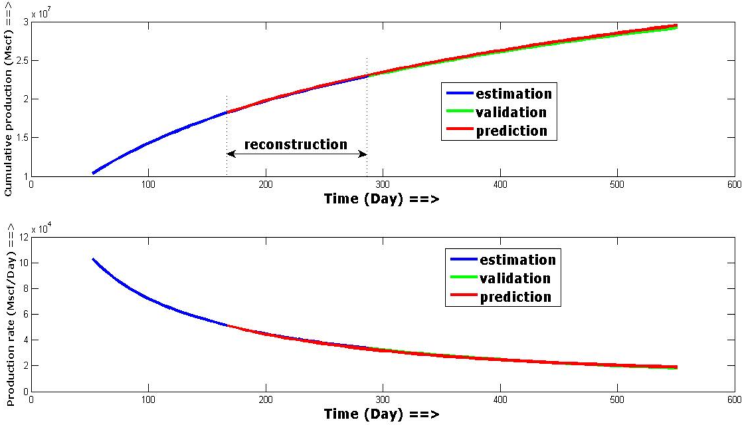
Predictions



Information from Dynamical Symmetry Portrait

- Given a point (Q_t, q_t) in the $Q - q$-*Phase Space*. then there is a special mapping $\Omega^i = (\Omega^i_Q, \Omega^i_q)$ relating the points associated with the same time parameter t on the U^i - and T^i decomposition of relation i :
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- Extensions
  - more extensive use of symplectic geometry
  - other perspectives (Riemannian Geometry)
- More information
  - Statistical and Applied Mathematics at Bath (**SAMBa**)
  - Presentation ('The Camera Physica') at SPE workshop in Galveston in November 2014
- Final Remark:
  - Production systems do not 'live' in the world of our daily experieces
  - The concern about production analytics tools, and lacking of new ideas results from igmoring this fact.